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Balancing Stability and Equity in Target Benefit Plans

A Novel Benefit Adjustment Mechanism

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Abstract

Target benefit plans, or collective defined contribution plans, aim to provide stable retirement income by pooling demographic and investment risks. However, balancing benefit stability with intergenerational equity remains a challenge, particularly in volatile markets. Building on the U.K. Royal Mail's collective defined contribution scheme, this paper proposes a novel benefit adjustment mechanism with parameter θ to better manage benefit volatility and intergenerational wealth transfers.

We introduce two risk measures – **benefit stability risk measure** and **wealth risk measure** – and a composite equity and stability index to evaluate trade-offs. Additionally, we analyze the impact of aggressive, balanced and conservative investment strategies on benefit outcomes. Through simulations, we demonstrate how the proposed mechanism achieves a fairer distribution of risks across generations. This study offers actionable insights for designing sustainable and equitable TBPs, contributing to the discourse on pension design in dynamic economic environments.

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1. Introduction

A target benefit plan (TBP), also known as a collective defined contribution plan, is an innovative pension design that combines elements of defined benefit and defined contribution pension plans. By pooling demographic and investment risks among participants, TBPs aim to provide stable retirement income. These plans adjust benefits based on the plan's financial position, reducing the need for frequent changes in employer contributions or reliance on external guarantees. However, a key challenge remains: balancing benefit stability with intergenerational fairness, particularly in volatile market conditions.

In the earlier study ***Balancing Act: Exploring Intergenerational Risk in Target Benefit Plans*** (Ma, 2024), the accumulation phase of the U.K. Royal Mail's collective defined contribution (RM-CDC) scheme was examined. The study analyzed how intergenerational risk-sharing operates under different market scenarios. The RM-CDC scheme uses a benefit adjustment mechanism that effectively smooths payouts, reducing income volatility for participants nearing retirement. However, this stability often comes at the cost of younger or mid-career participants, who bear a disproportionate share of the adjustments. This raises significant concerns about intergenerational equity.

This paper builds on the previous foundational analysis by exploring the RM-CDC benefit adjustment mechanism in greater depth, with a focus on the accumulation phase of TBPs. We examine its impact on benefit stability and equity and address its limitations by introducing a novel adjustment mechanism. Central to this new approach is the adjustment parameter θ, which provides enhanced control over benefit payout volatility and wealth transfers across generations. Our goal is to achieve a more effective balance between stability and equity.

To assess the effectiveness of the proposed mechanism, we introduce two key risk measures:

1. Benefit stability risk measure: Quantifies the volatility of benefit payouts.
2. Wealth risk measure: Evaluates the extent of intergenerational wealth transfers.

Additionally, we define the equity and stability index, a composite measure designed to determine the optimal adjustment parameter for balancing these competing priorities. Together, these metrics provide a robust framework for analyzing the trade-offs inherent in TBP design.

The paper also explores how different investment strategies influence benefit outcomes under the proposed adjustment mechanism. We examine alternative investment approaches – aggressive, balanced and conservative strategies – to provide a comprehensive understanding of how investment decisions impact both benefit stability and equity. By integrating these findings, the paper offers actionable insights for designing TBPs that align with the plan's objectives and the risk tolerance of its stakeholders.

Organization of the paper

The paper is structured as follows. Section 2 reviews the key trade-offs between benefit stability and equity in TBPs, focusing on the RM-CDC scheme's benefit adjustment mechanism. Section 3 introduces the proposed adjustment mechanism with the parameter θ, detailing its formulation and advantages. Section 4 presents the risk measures (ϕ) and (ψ) and the equity and stability index (λ) as means for evaluating and optimizing adjustment mechanisms. Section 5 focuses on the optimization process for determining the ideal value of θ, leveraging simulations to analyze outcomes. Section 6 evaluates the impact of alternative investment strategies on benefit outcomes, highlighting the trade-offs associated with different asset mixes. Finally, Section 7 synthesizes the findings, discussing their practical implications for TBP design, followed by concluding remarks and avenues for future research in Section 8.

2. Navigating the trade-off: Stability and equity in target benefit plans

The accumulation phase of a TBP presents a significant challenge in achieving both benefit stability and equity across generations. In *Balancing Act* (Ma, 2024), we examined how benefit smoothing and wealth transfers within a model TBP plan unfold under various market conditions – unfavourable, consistent and favourable.

Building on that foundation, this study evaluates the efficacy of the RM-CDC framework, a type of TBP, in balancing these objectives: ensuring stable retirement benefits while equitably distributing investment outcomes across generations.

Our analysis leverages the TBP and individual defined contribution (IDC) plan models and investment scenarios detailed in *Balancing Act* (Ma, 2024). Appendix A outlines the main features of the model TBP plan, including its membership profile and key valuation assumptions. The plan employs the RM-CDC benefit adjustment mechanism to maintain financial equilibrium and ensure stable payouts. This mechanism achieves stability through periodic adjustments to future escalation rates on participants' accrued benefits. While this approach reduces income volatility for members nearing retirement, it places a greater burden on mid-career participants, whose benefits are affected by adjustments to both accrued and future service benefits.

Numerical illustrations presented throughout this paper are based on the above TBP and individual defined contribution models.

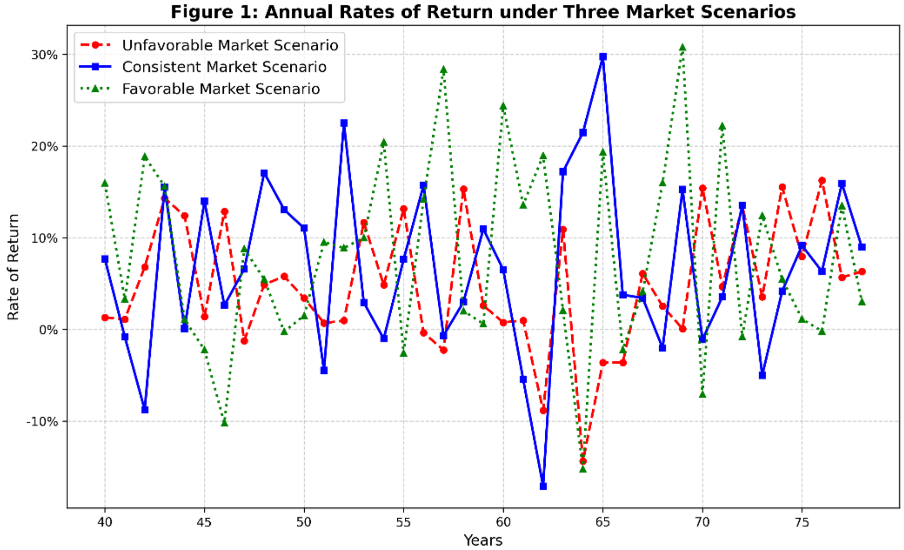
2.1 Investment scenarios

The fund for the model TBP plan is assumed to invest in a balanced portfolio, with 50% in equities and 50% in bonds. To provide a comprehensive analysis, three distinct investment scenarios were selected from 1,000 simulations generated using the economic scenario generator described in Appendix B. These scenarios cover the period between time 40 and time 80:

- **Unfavourable markets:** This scenario represents the 25th percentile of simulated outcomes, with a geometric average return of 4.6% per annum. It reflects persistently weak investment performance and adverse economic conditions.
- **Consistent markets:** Representing the median outcome, this scenario assumes a geometric average return of 6.1% per annum, consistent with long-term expectations under stable economic conditions.
- **Favourable markets:** Reflecting the 75th percentile of outcomes, this scenario assumes a geometric average return of 7.5% per annum, capturing periods of sustained strong market performance.

These scenarios span a range of potential market conditions, enabling a thorough examination of how benefit stability and intergenerational equity respond to diverse economic environments. Figure 1 depicts the annual rates of return under the three market scenarios.

Figure 1: Annual Rates of Return under Three Market Scenarios



2.2 Evaluation measures

Two key metrics, the benefit payout ratio (BPR) and the collective wealth ratio (CWR), enable an evaluation of both benefit stability and intergenerational equity within the model TBP plan.

The benefit payout ratio

The BPR measures the benefit paid at retirement as a proportion of the targeted retirement benefit under the plan. This metric is critical for assessing the adequacy and stability of benefit payouts. For each generation, the BPR is calculated at retirement under varying investment scenarios, with logarithmic transformations applied to capture proportional changes and enable standardized comparisons across generations.

To assess benefit stability, we compute the standard deviation of log changes in BPR,[1] denoted as $\sigma\_ln(\Delta BPR)$, across generations. A lower standard deviation indicates that the benefit levels are more consistent across time and less influenced by market volatility or demographic shocks. This consistency fosters participant confidence and underscores the robustness of TBP design. Stability in BPR is also critical for evaluating the comparative effectiveness of alternative benefit adjustment mechanisms.

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[1] The log change in the BPR is the logarithm of the ratio between two consecutive BPR values. Let BPR_k denote the BPR for generation $\#k$, the log change between two consecutive values, BPR_k and BPR_{k-1} $(k > 1)$, is calculated as:

$$Log\ Change\ =\ ln(\frac{BPR_k}{BPR_{k-1}})$$

The log change represents the relative change between two consecutive observations. A positive log difference indicates an increase, while a negative log difference indicates a decrease. A log difference of 0 indicates no change.

The collective wealth ratio

The CWR assesses the collective wealth of participants within a TBP, defined as the ratio of the fund's assets to the accumulated contributions of all participants in the plan, adjusted for investment returns. A CWR of 1.0 represents perfect balance, where the plan's assets are exactly sufficient to cover the participants' accumulated contributions. By definition, the CWR for individual defined contribution plans is always equal to 1.0. Deviations from this benchmark indicate intergenerational wealth transfers:

- **CWR > 1.0** reflects an excess, where investment or demographic gains from prior generations benefit current participants.
- **CWR < 1.0** reflects a shortfall, where current participants effectively subsidize prior generations by bearing the burden of earlier losses.

By analyzing the distribution of simulated CWR values, we evaluate how the adjustment mechanism under the model TBP plan influences intergenerational fairness.

2.3 Analysis of the model plan

Benefit stability

The model TBP plan demonstrates robust stability in benefit payouts, as evidenced by significantly lower variability in the log changes of BPR across generations #1 to #40, over the period between time 40 and time 80. For example:

- **Unfavourable markets:** The standard deviation of log changes in BPR is notably lower for the TBP plan (2.4%) compared to the individual defined contribution plan with the same contribution rate (7.0%).
- **All market scenarios:** This trend holds across different market conditions, highlighting the robustness of RM-CDC adjustments in mitigating benefit payment volatility.

Intergenerational wealth transfers

An analysis of CWR at time 80 – when the last participant from generations #1 to #40 exits the plan – reveals significant deviations from the ideal balance (CWR = 1.0), indicating wealth transfers under the TBP plan:

- **Unfavourable markets:** A CWR of 0.849 reflects a shortfall, with current participants bearing the cost of stabilizing benefits for earlier generations (#1 to #40).
- **Consistent markets:** A near-balanced CWR of 1.01 suggests minimal wealth transfers under stable conditions.
- **Favourable markets:** A sharp rise in CWR to 1.308 shows that earlier generations disproportionately contribute to future participants' gains.

These findings underscore a trade-off: while the plan prioritizes benefit stability, it does so at the expense of intergenerational equity, varying with market conditions.

The case for balanced adjustment mechanisms

While the RM-CDC framework effectively smooths benefit payouts, its heavy reliance on intergenerational wealth transfers underscores the need for more balanced adjustment mechanisms. Achieving the right balance between stability and equity is crucial for the long-term sustainability of TBPs. This calls for innovative mechanisms that adapt to diverse market conditions while aligning participant outcomes with the core objectives of these plans.

3. Exploring balanced benefit adjustment mechanisms

3.1 RM-CDC adjustment

The benefit adjustment mechanism under the model TBP plan is grounded in an annual actuarial valuation that assesses the plan's financial position. The balance sheet for the plan at a given time t (subscript t suppressed) is set out in Table 1 below.

Table 1: Main entries on the valuation balance sheet of the model plan

Assets	Liabilities
Value of fund assets $(\boldsymbol{F})$	Past service liability for members (PSL)
Present value of future contributions $(\boldsymbol{PVFC})$	Future service liability for members (FSL)
Total assets $(\mathbf{1}) = \ \boldsymbol{F} \ + \ \boldsymbol{PVFC}$	Total liabilities $(2) = \ PSL \ + \ FSL$
Funding Excess (Deficit) = (1)-(2) $= \boldsymbol{F} + \boldsymbol{PVFC} - \boldsymbol{PSL} - \boldsymbol{FSL}$	

The formulas for calculating PSL, FSL and PVFC for the model plan are detailed in Appendix C of *Balancing Act* (Ma, 2024).

When a funding deficit or excess arises, the plan adjusts the escalation rate for accrued benefits to address the imbalance. While this approach ensures financial stability, its impacts vary across participant cohorts (Ma, 2024):

- Mid-career participants are most affected, as they have accrued significant benefits and still anticipate substantial future service benefits.
- Older participants nearing retirement experience less volatility due to their shorter remaining membership in the plan.
- Younger participants are relatively less impacted, given their small accrued benefits.

While the adjustment mechanism effectively stabilizes benefit payouts, it does so at the expense of intergenerational equity. The adjustments required to address funding imbalances often lead to wealth transfers across generations, creating disparities in how these imbalances are distributed.

3.2 Proposal for a balanced adjustment mechanism

To address the trade-offs inherent in the RM-CDC mechanism, we propose a more balanced approach that adjusts benefits for both past and future service. By distributing the effects of funding surpluses or deficits across all participants, this approach reduces the inequities borne by specific cohorts, particularly mid-career participants.

The funding excess (deficit), denoted as FE, is derived from the actuarial balance sheet as follows:

$$FE = F + PVFC - PSL - FSL$$

We introduce an adjustment parameter θ (where $0 \leq \theta \leq 1$) to apportion FE between past and future service:

- **Past service adjustments**: $\theta \cdot FE$ is allocated to adjust benefits already accrued by participants.
- **Future service adjustments**: $(1 - \theta) \cdot FE$ is applied to future service benefits, spreading the impact across longer time horizons.

The benefit adjustment factors are calculated as follows:

- **Past service**: $h_t^{PS} = 1 + \theta \cdot FE/PSL$
- **Future service**: $h_t^{FS} = 1 + (1 - \theta) \cdot FE/FSL$

The accrued benefit for participant j is updated from time $t - 1$ to time t using these factors as follows:

$$AB_t^j = h_t^{PS} \cdot AB_{t-1}^j(1 + \delta) + h_t^{FS} \cdot CB_t^j,$$

where AB_t^j is the accrued benefit at time t, δ is the target indexing rate and CB_t^j is the current-year benefit accrual under the plan formula.

3.3 Advantages of the proposed mechanism

The proposed benefit adjustment mechanism offers two key advantages:

1. **Enhanced equity**
   This mechanism ensures a more equitable distribution of funding adjustments across generations by allocating them to both past and future service benefits. Unlike RM-CDC-like adjustments, it reduces the financial burden on any single cohort, particularly mid-career participants.
2. **Improved sustainability**
   By incorporating adjustable parameters, the mechanism introduces flexibility to align funding adjustments with the plan's long-term financial sustainability, ensuring it remains robust over time.

Illustrative example

Consider a TBP with a funding deficit of $10 million, a past service liability (PSL) of $100 million, and a future service liability (FSL) of $150 million. Using $\theta = 0.5$, 50% of the deficit ($5 million) is applied to reduce past service benefits, and the remaining 50% ($5 million) is applied to reduce future service benefits. The benefit adjustment factors are:

- $h_t^{PS} = 1 - \frac{5}{100} = 0.95,$ or a 5% reduction to past service benefits, and
- $h_t^{FS} = 1 - \frac{5}{150} = 0.967$, or a 3.3% reduction to future service benefits.

This approach distributes the amount of the deficit equally between the past and future service, fostering stability while maintaining relative fairness, as demonstrated in the next section.

3.4 Comparison between the proposed adjustment and the RM-CDC adjustment

This section evaluates the differences in BPRs between the individual defined contribution plan, the TBP with the RM-CDC adjustment, and the TBP with adjustment parameter (TBP(0.5)) under the three investment scenarios described in Section 2.1:

- unfavourable markets
- consistent markets
- favourable markets

The findings are illustrated in Figures 2(a), (b) and (c) and Table 2, below.

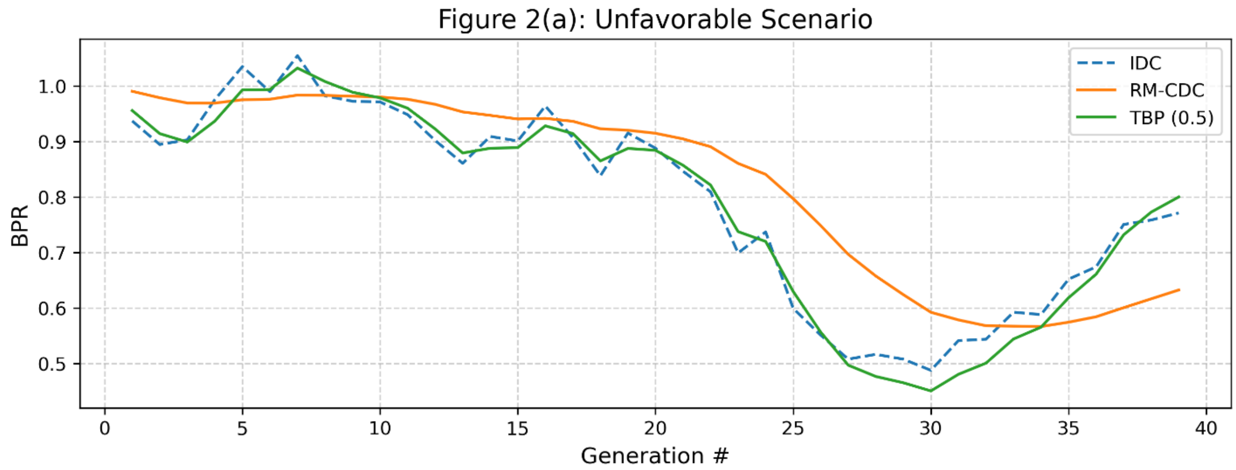
Figure 2(a): Unfavorable Scenario
IDC
RM-CDC
TBP (0.5)
BPR
Generation #

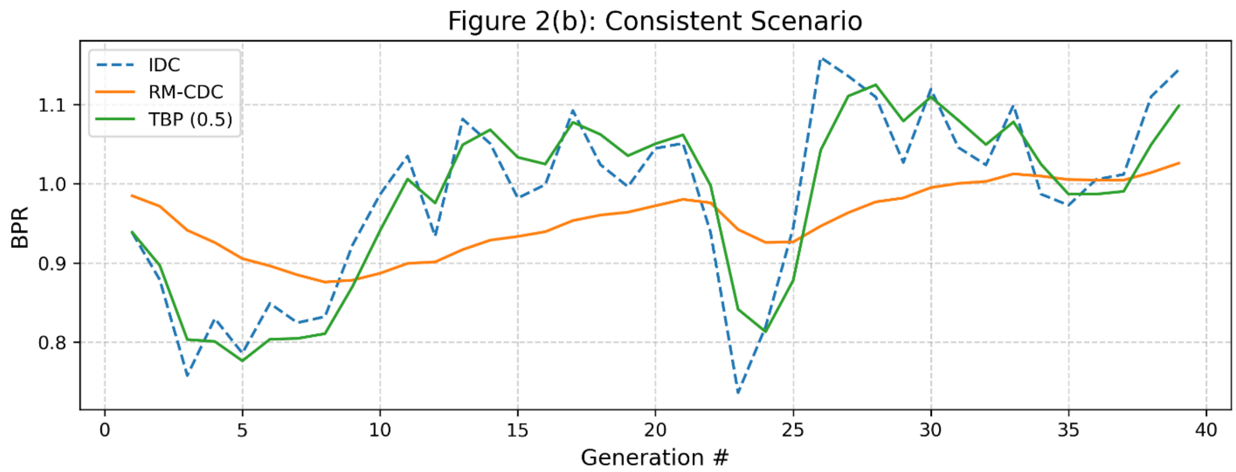
Figure 2(b): Consistent Scenario
IDC
RM-CDC
TBP (0.5)
BPR
Generation #

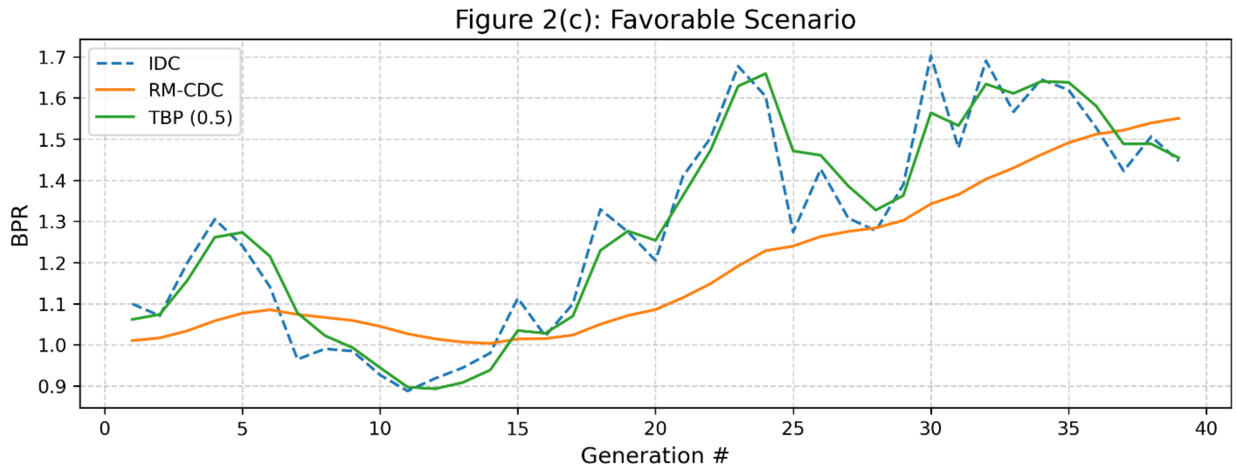
Figure 2(c): Favorable Scenario
IDC
RM-CDC
TBP (0.5)
BPR
Generation #

Table 2: Summary of key statistics

	Unfavourable markets			Consistent markets			Favourable markets		
	IDC	RM-CDC	TBP (0.5)	IDC	RM-CDC	TBP (0.5)	IDC	RM-CDC	TBP (0.5)
$\boldsymbol{\mu\_ln}(\Delta \boldsymbol{BPR})$	-0.61%	-1.17%	-0.54%	0.55%	0.09%	0.36%	0.93%	1.12%	0.95%
$\boldsymbol{\sigma\_ln}(\Delta \boldsymbol{BPR})$	6.95%	2.41%	5.74%	8.98%	1.33%	5.87%	9.83%	1.35%	6.17%
$\boldsymbol{CWR}$	1.000	0.849	1.028	1.000	1.010	0.995	1.000	1.308	0.981

Observations from Figures 2(a), (b) and (c)

Figures 2(a), (b) and (c) depict the evolution of BPRs across generations #1 to #40. Key observations include:

- **Smoother adjustments with RM-CDC**: The RM-CDC adjustment results in smoother changes in BPRs compared to TBP(0.5). However, the levels of BPRs under RM-CDC deviate significantly from those under IDC, highlighting its focus on stability rather than alignment with the individual defined contribution plan.
- **Closer tracking with TBP(0.5)**: The BPRs under TBP(0.5) align more closely with those of the individual defined contribution plan, demonstrating a balance between tracking IDC benefits and smoothing fluctuations. This indicates that TBP(0.5) provides moderate benefit stability while maintaining a reasonable resemblance to IDC outcomes.

Observations from Table 2

Table 2 presents the mean and standard deviation of the log changes in BPRs (denoted as $\mu\_ln(\Delta BPR)$ and $\sigma\_ln(\Delta BPR)$, respectively) and the CWRs at time 80 across the three plans under different market conditions.

Key insights include:

1. **Stability vs. equity**: The RM-CDC adjustment achieves the lowest $\sigma\_ln(\Delta BPR)$ values across all scenarios, indicating the most stable benefit payments. However, this comes at the cost of intergenerational equity, as evidenced by the CWR values, which deviate significantly from 1 in unfavourable and favourable market conditions.
2. **Moderation with TBP(0.5)**: TBP(0.5) demonstrates moderately lower $\sigma\_ln(\Delta BPR)$ compared to the individual defined contribution plan, reflecting modest stability in benefit payments. Its CWR values remain closer to 1, suggesting limited intergenerational wealth transfers.
3. **Stakeholder considerations**: The RM-CDC adjustment may be unsuitable for stakeholders prioritizing intergenerational equity. TBP(0.5), by contrast, offers a more balanced approach, achieving moderate stability with less inequity across generations.

These results highlight the trade-offs inherent in choosing an adjustment mechanism. While the RM-CDC mechanism helps to achieve greater stability in benefit payments, the TBP(0.5) mechanism offers a more balanced approach, effectively addressing both benefit stability and intergenerational equity.

The proposed mechanism advances the design of benefit adjustments in TBPs by integrating considerations of equity and stability. It reduces dependence on intergenerational wealth transfers and establishes a more sustainable framework for managing benefit volatility.

In the following sections, we examine how variations in the adjustment parameter θ influence benefit stability and intergenerational equity. We also identify the optimal parameter values that achieve a more balanced outcome, aligning with the plan's objectives.

4. Introducing the equity and stability index

In the design and management of TBPs, achieving benefit stability and intergenerational equity are core objectives. To evaluate the efficacy of various adjustment parameters in meeting these objectives, we introduce the concept of the equity and stability index (λ). This index serves as a comprehensive measure to balance benefit stability and intergenerational equity under a wide range of market conditions.

4.1 Simulation framework

Our analysis is grounded in a simulation framework that models 1,000 investment scenarios for each adjustment parameter under consideration (see Section 2.1). This approach captures plan performance across diverse market conditions, enabling the evaluation of the following metrics:

1. **Standard deviations of log changes in BPR ($\boldsymbol{\sigma\_ln}(\Delta \boldsymbol{BPR})$)**: The distribution of these standard deviations provides a visual representation of benefit stability over time. Lower values of $\sigma\_ln(\Delta BPR)$ indicate greater stability, as they reflect reduced variability in benefit payouts due to market fluctuations. Key statistics such as the mean and standard deviation are calculated to compare adjustment parameters. These results highlight how different parameters mitigate or amplify the effects of market volatility on participants' benefits, making it an essential metric for plan evaluation.

2. **Distributions of simulated CWRs**: To assess intergenerational equity, we examine the CWRs and their distributions. Statistics like the mean and standard deviation offer insights into the fairness of wealth transfers among generations. Adjustment parameters producing tighter CWR distributions with smaller standard deviations (σ_CWR) are preferred for minimizing unintended wealth imbalances. By focusing on the spread and central tendency of these distributions, we can gauge the degree to which each parameter promotes equity across different cohorts of participants.

4.2 Comparative analysis

Figures 3 and 4 illustrate the kernel density estimate (KDE)[2] plot of the distribution of $\sigma\_ln(\Delta BPR)$ and CWR, respectively, for adjustment parameters ranging from 0 to 1. Below are the means and standard deviations of these distributions.

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[2] Kernel density estimation is a non-parametric statistical technique used to estimate the probability density function of a random variable based on a finite sample. It smooths data by overlaying individual distributions – commonly Gaussian kernels – on each data point and summing them to produce a

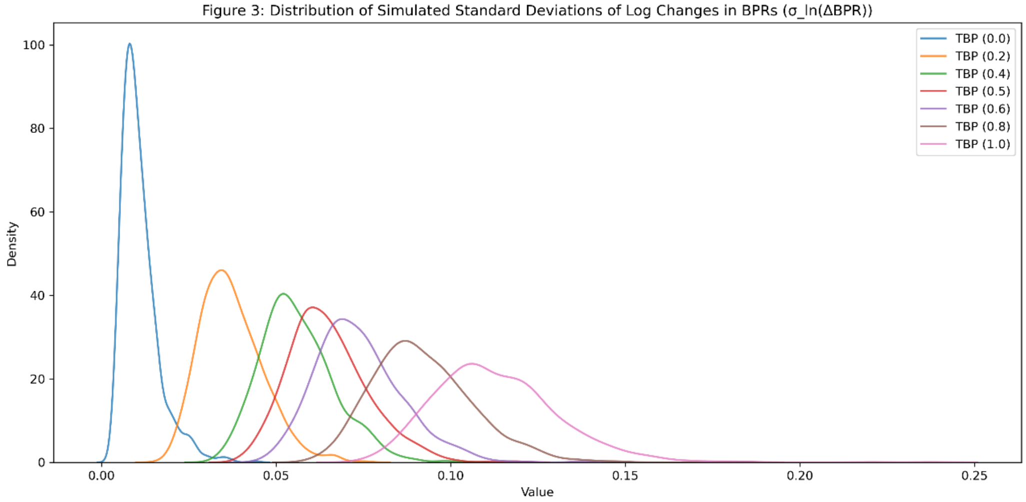
Standard deviation of log changes in benefit payout ratios ($\sigma\_ln(\Delta BPR)$)

	Adjustment parameter						
	$\theta = 0$	$\theta = 0.2$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.8$	$\theta = 1.0$
Mean	0.011	0.037	0.056	0.065	0.074	0.092	0.113
Standard deviation	0.005	0.009	0.011	0.011	0.012	0.015	0.018

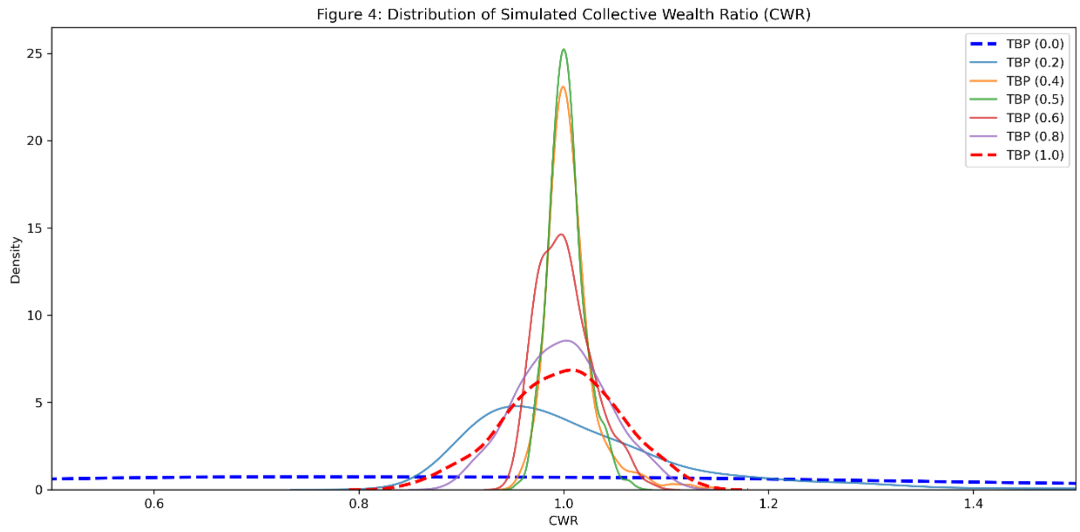
Collective wealth ratio (CWR)

	Adjustment parameter						
	$\theta = 0$	$\theta = 0.2$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.8$	$\theta = 1.0$
Mean	1.073	1.019	1.006	1.003	1.001	0.999	0.998
Standard deviation	0.610	0.115	0.024	0.018	0.027	0.044	0.056

Figure 3: Distribution of Simulated Standard Deviations of Log Changes in BPRs (σ_ln(ΔBPR))



continuous curve. The bandwidth parameter, which determines the width of the kernel, plays a critical role in balancing bias and variance in the estimate. Smaller bandwidths capture more detail but may lead to overfitting, while larger bandwidths smooth the distribution at the cost of resolution. KDE is particularly effective for visualizing and analyzing the underlying structure of complex data distributions, making it a valuable tool in assessing metrics like $\sigma\_ln(\Delta BPR)$ and CWR.



Key observations

- **Benefit stability:** Lower adjustment parameters correspond to lower mean and standard deviation of $\sigma\_ln(\Delta BPR)$, indicating reduced benefit volatility. This result underscores the effectiveness of smaller adjustment parameters in stabilizing payouts.
- **Intergenerational equity:** Parameters in the range of 0.4 to 0.6 produce CWR distributions with smaller spreads compared to other parameters such as 0.2 or 0.8, suggesting a better balance of wealth transfers. The findings highlight the nuanced relationship between adjustment parameter selection and equity outcomes.

The simulation results reveal trade-offs inherent in TBP design. Mechanisms prioritizing benefit stability may compromise equity, and vice versa. For example, highly stable benefits may come at the cost of larger deviations in wealth transfers, potentially disadvantaging certain generations. Conversely, mechanisms that closely maintain CWRs around 1.0 may exhibit greater variability in benefit payouts, reducing predictability for participants.

4.3 Benefit and equity risk measures

Two risk measures are defined to quantify the dual objectives of benefit stability and intergenerational equity:

- **Benefit stability risk measure (ϕ):** The expected value (mean) of $\sigma\_ln(\Delta BPR)$, denoted as $\mathbb{E}(\sigma\_ln(\Delta BPR))$, is used to gauge benefit stability. This measure captures the average volatility in benefit payouts, impacting predictability and participant confidence. A lower value of ϕ indicates greater success in achieving stable benefits, which is essential for plan members relying on predictable income streams.
- **Wealth risk measure (ψ):** The standard deviation of CWR, denoted as σ_CWR, is used to evaluate intergenerational equity. This measure reflects the variability in wealth transfers, with smaller values indicating greater fairness and balance across generations. It provides a direct quantification of how well a parameter reduces disparities in the allocation of financial resources over time.

4.4 Equity and stability index

The equity and stability index is defined as a weighted average of ϕ and ψ: $\lambda = w \cdot \phi + (1 - w) \cdot \psi$, where w lies between 0 and 1. Here, w represents the benefit stability weight, and its complement, $1 - w$, represents the equity weight. By adjusting w, the index allows stakeholders to prioritize stability or equity:

- For greater emphasis on benefit stability, set w closer to 1.
- For greater emphasis on equity, set w closer to 0.
- For a balanced approach, set w = 0.5.

The optimal adjustment parameter (θ) is the one that minimizes λ for the given weight w. This approach provides a systematic method to evaluate trade-offs and select a parameter aligned with the plan's goals.

4.5 Practical implications

The equity and stability index (λ) provides a quantitative tool for balancing the benefit stability and equity objectives, guiding plan sponsors, policymakers and participants toward the most suitable adjustment mechanisms. This index can be integrated into decision-making processes to evaluate and compare the outcomes of different plan designs under varied market conditions. By leveraging the insights from the simulation framework, stakeholders can:

- Identify optimal adjustment parameters that align with the desired balance of stability and equity.
- Tailor plan designs to specific demographic or economic contexts, ensuring the long-term sustainability of TBPs.
- Communicate trade-offs effectively to participants, enhancing transparency and trust.

Through this framework, TBPs can better align their designs with the dual goals of stability and fairness, ensuring sustainable outcomes across generations.

5. Designing an optimal benefit adjustment strategy

This section outlines the methodology for determining the optimal adjustment parameter (θ) to balance benefit stability and intergenerational equity in TBPs. The optimization process minimizes the equity and stability index (λ), defined as:

$$\lambda = w\phi + (1 - w)\psi,$$

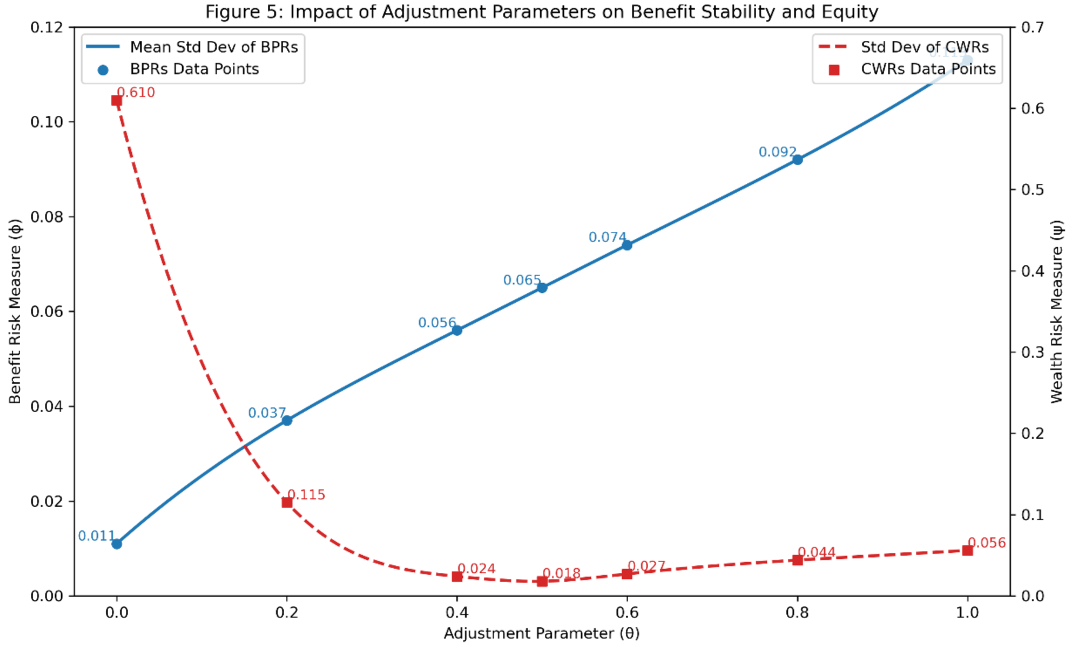
where

- ϕ represents the benefit stability risk measure;
- ψ represents the wealth risk measure; and
- w is the benefit stability weight.

The objective is to adjust θ to achieve the lowest possible value of λ, reflecting trade-offs between these competing objectives.

5.1 Impact of adjustment parameters on benefit outcomes

Figure 5 illustrates the relationship between the adjustment parameter θ and the risk measures of benefit stability (ϕ) and wealth (ψ) for the model TBP described in Appendix A. Cubic spline interpolation is used to smooth the datasets of ϕ and ψ, resulting in continuous curves.



Key observations from the plot include:

- As θ increases, the benefit stability risk measure (ϕ) progressively rises, indicating greater risk to benefit stability over time.
- The wealth risk measure (ψ) remains relatively high for low adjustment parameters ($\theta \leq 0.2$) but decreases significantly as θ increases.

A low adjustment parameter (θ) requires younger participants and future generations to bear most of the burden related to prior funding imbalances. Conversely, a higher θ shifts more of the burden to older participants. Specifically, a zero value of θ places the funding imbalance predominantly on younger participants. An intermediate range ($\theta = 0.4 - 0.6$) appears to balance benefit stability and equity more effectively.

For comparison, in the individual defined contribution plan, the benefit stability risk measure (ϕ) is 0.092, and the equity risk measure (ψ) is zero. This suggests that for the TBP, when $\theta > 0.8$, the increased volatility of benefit payments provides no added advantage in preserving intergenerational equity.

Further analysis will explore the implication of varying θ for plan design.

5.2 Optimization methodology

The process to find the optimal adjustment parameter (θ) that balances benefit stability and equity involves the following steps:

1. **Data preparation:**
    - Define an array of adjustment parameters between 0 and 1, say, θ = 0.0, 0.2, 0.4, 0.5, 0.6, 0.8 and 1.0.
    - Run simulations on the model TBP plan to compute the corresponding benefit stability risk measures (ϕ) and wealth risk measures (ψ).
2. **Interpolation:**
    - Use cubic spline interpolation to create continuous functions for ϕ and ψ, as functions of θ.
3. **Objective function:**
    - Minimize the equity and stability index (λ), defined as a weighted sum of the interpolated values of ϕ and ψ.
4. **Optimization:**
    - Perform bounded optimization for different benefit stability weights, such as 0.0, 0.2, 0.4, 0.5, 0.6, 0.8, and 1.0.

Table 3, below, summarizes the results. The Python code used for the calculations is provided in Appendix C.

Table 3: Optimal adjustment parameters for different weight combinations

Stability weight (w)	Adjustment parameter (θ)	Benefit stability risk measure (ϕ)	Wealth risk measure (ψ)
0.0	0.493	0.064	0.018
0.2	0.482	0.063	0.018
0.4	0.461	0.062	0.019
0.5	0.438	0.060	0.020
0.6	0.371	0.054	0.028
0.8	0.300	0.047	0.044
1.0	0.000	0.011	0.611

For example, if the benefit stability weight w is 0.5, the optimal adjustment parameter is 0.438. Simulations using this parameter yield $\phi = 0.06$ and $\psi = 0.019$, closely matching the optimization results shown above and demonstrating the robustness of the method.

It is also noteworthy that adjustment parameters between 0.4 and 0.5 result in similar risk levels for benefit stability and equity. However, achieving greater benefit stability would require the adoption of a lower adjustment parameter, which entails accepting higher levels of wealth transfers across generations.

5.3 A special adjustment parameter

In the preceding analysis, the adjustment parameter (θ) was assumed to be fixed throughout the life of the plan. This section introduces a varying adjustment parameter defined as:

$$\theta = \frac{PSL_t}{PSL_t + FSL_t},$$

where θ varies over time t based on the relative proportions of past service liabilities (PSL) and future service liabilities (FSL). This formulation ensures that θ reflects the evolving funding structure of the plan.

Benefit adjustment factors

Using this adjustment parameter, the benefit adjustment factor for past service is given by:

$$h_t^{PS} = 1 + \frac{\theta(F_t + PVFC_t - PSL_t - FSL_t)}{PSL_t} = 1 + \frac{(F_t + PVFC_t - PSL_t - FSL_t)}{PSL_t + FSL_t} = \frac{F_t + PVFC_t}{PSL_t + FSL_t}$$

where

- F_t represents the fund value at time t;
- $PVFC_t$ is the present value of future contributions at time t; and
- PSL_t and FSL_t denote past and future service liabilities at time t, respectively.

Similarly, the benefit adjustment factor for future service is given by:

$$h_t^{FS} = \frac{F_t + PVFC_t}{PSL_t + FSL_t}.$$

The special adjustment parameter yields the same benefit adjustment factor for both past and future service, which is equal to the funded ratio of the plan, i.e., the ratio of $F_t + PVFC_t$ to $PSL_t + FSL_t$ at any time t.

Simulation results and observations

Simulations conducted on the model TBP using the special adjustment parameter yield the following results:

- benefit stability risk measure (ϕ) = 0.070
- equity risk measure (ψ) = 0.024

While this special parameter provides a tailored adjustment approach, its performance is less efficient compared to fixed adjustment parameters in the range of θ = 0.4 to 0.5, as analyzed in Section 5.2. Specifically, the fixed parameter achieves a better balance between benefit stability and intergenerational equity, as reflected in lower values of both ϕ and ψ.

This result suggests that while dynamically adapting θ can align benefit adjustments with the plan's funding structure over time, it may not always optimize the trade-off between stability and equity. Further research could explore refinements to this parameter to enhance its efficiency.

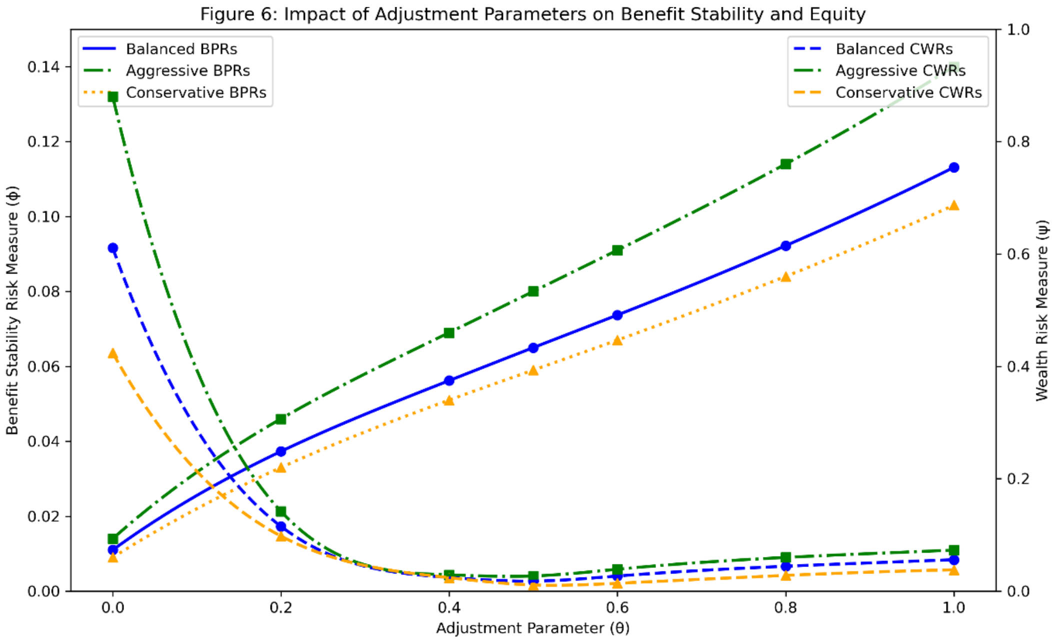
6. Evaluating alternative investment strategies in target benefit plans

In the previous section, we analyzed the outcomes of a balanced investment strategy with a 50/50 allocation between equities and bonds. This section extends the analysis by exploring the benefit outcomes under two contrasting investment strategies: an aggressive strategy with a higher equity allocation and a conservative strategy with a lower equity allocation. These approaches offer differing methods of managing risk and achieving retirement income objectives, providing valuable insights for plan decision makers.

The aggressive strategy allocates 70% of the portfolio to equities and 30% to bonds, resulting in an expected annual return, net of expenses, of 6.75%. This return also determines the valuation interest rate. By contrast, the conservative strategy allocates 30% to equities and 70% to bonds, yielding an expected annual return of 5% and a corresponding valuation interest rate of 5%. For both strategies, the contribution rate is fixed at 10.83%, as outlined in Appendix A, ensuring consistency in the model inputs.

For the TBP model, these strategies produce distinct outcomes in terms of target benefit rates and income replacement ratios. Under the aggressive strategy, the target benefit rate is 2.09%, corresponding to a target retirement income ratio of 71%, a significant increase from the 56.6% observed under the balanced strategy. Conversely, the conservative strategy yields a target benefit rate of 1.21%, resulting in a target retirement income ratio of 41.1%.

To evaluate the implications of these strategies, we examine two key risk measures: the benefit stability risk measure (ϕ) and the wealth risk measure (ψ). Figure 6 illustrates the trends for these measures across TBP plans under each strategy. For the individual defined contribution plan, the benefit stability risk measures under the aggressive and conservative strategies are 0.111 and 0.087, respectively, compared to 0.092 under the balanced strategy. These variations underscore the differing risk profiles inherent in each investment approach.



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The analysis reveals that the aggressive strategy shifts both risk measure curves upward from those of the balanced strategy, indicating heightened risks to benefit stability and intergenerational equity. In contrast, the conservative strategy shifts these curves downward, reflecting reduced risks in both dimensions. These findings highlight the inherent trade-offs in selecting an investment strategy, where higher returns often come with increased volatility and equity risk.

Tables 4 and 5 summarize the optimal adjustment parameters (θ) and their associated risk measures across different benefit stability weights (w). For the aggressive strategy, the optimal adjustment parameter decreases as the weight assigned to benefit stability increases, ranging from 0.473 at w = 0.0 to 0.000 at w = 1.0. This pattern aligns with the observed decrease in benefit stability risk measure (ϕ) and corresponding increase in wealth risk measure (ψ). The conservative strategy exhibits a similar trend, albeit with generally lower risk levels.

For instance, at w = 0.5, the optimal adjustment parameter for the aggressive strategy is 0.361, compared to 0.488 for the conservative strategy and 0.438 for the balanced strategy.

Table 4: Aggressive investment strategy

Stability weight (w)	Adjustment parameter (θ)	Benefit stability risk measure (ϕ)	Wealth risk measure (ψ)
0.0	0.473	0.077	0.026
0.2	0.452	0.075	0.027
0.4	0.376	0.066	0.030
0.5	0.361	0.065	0.032
0.6	0.345	0.063	0.034
0.8	0.298	0.058	0.048
1.0	0.000	0.014	0.880

Table 5: Conservative investment strategy

Stability weight (w)	Adjustment parameter (θ)	Benefit stability risk measure (ϕ)	Wealth risk measure (ψ)
0.0	0.524	0.061	0.010
0.2	0.514	0.060	0.011
0.4	0.499	0.059	0.011
0.5	0.488	0.058	0.012
0.6	0.466	0.056	0.014
0.8	0.286	0.041	0.050
1.0	0.000	0.009	0.424

These findings illustrate the trade-offs involved in adopting each investment strategy. The aggressive strategy offers the potential for higher retirement income, as evidenced by its elevated target benefit rate and income replacement ratio. However, this advantage comes with increased risks, including greater benefit volatility and reduced intergenerational equity. In contrast, the conservative strategy prioritizes risk mitigation, achieving lower levels of benefit and equity risk but at the cost of reduced retirement income potential.

This trade-off underscores the importance of aligning investment strategy decisions with the plan's objectives and stakeholder preferences. By carefully balancing these competing factors, decision makers can select an approach that best meets the long-term goals of the plan while managing risks effectively.

7. Implications for the design of target benefit plans

The design elements of TBPs, including the benefit adjustment mechanism and investment strategy, are essential in determining benefit outcomes. An in-depth analysis of these elements, as previously presented, reveals significant implications for the design and administration of TBPs. Understanding and optimizing these design elements can help plans achieve their primary goals of fair distribution of retirement wealth and benefit stability.

7.1 Balancing stability and equity

TBPs aim to balance the competing objectives of benefit stability and intergenerational equity, making their design choices critical for ensuring the long-term sustainability and fairness of the plan. The plan may rely on mechanisms like the RM-CDC framework to stabilize benefit payouts, but this stability comes at a cost. These mechanisms can result in significant intergenerational wealth transfers, where current participants bear the cost of stabilizing benefits for earlier cohorts in unfavourable market conditions. Conversely, in favourable markets, surpluses may disproportionately benefit future generations, highlighting the sensitivity of equity outcomes to market dynamics.

The trade-off between stability and equity is particularly evident in the analysis of adjustment parameters. Lower adjustment parameters emphasize stability and reduce benefit volatility, but they may impose greater burdens on younger participants. Higher adjustment parameters shift the burden to older participants, potentially causing greater fluctuations in benefit payouts. A balanced approach, such as using intermediate adjustment parameters, offers a potential pathway to achieving a more equitable distribution of risks and rewards across generations.

7.2 Impact of investment strategies

The choice of investment strategy also significantly influences TBP outcomes. Aggressive strategies, with higher equity content, can increase retirement benefits but also elevate risks to benefit stability and equity. On the other hand, conservative strategies provide greater stability and reduced risk at the expense of lower benefit levels. These trade-offs underscore the importance of aligning investment strategies with the plan's objectives and the risk tolerance of its stakeholders.

7.3 Sustainability and flexibility

The dynamic interplay between economic conditions, adjustment mechanisms and investment strategies necessitates a forward-looking approach to TBP design. Mechanisms that adapt to varying market conditions, such as dynamic adjustment parameters or balanced benefit adjustments, can enhance sustainability by mitigating the adverse effects of extreme market conditions. Moreover, incorporating flexibility in plan rules and adjustment mechanisms allows TBPs to evolve in response to changing demographic and economic landscapes, ensuring their relevance and effectiveness over time.

For instance, dynamic adjustment parameters could prioritize wealth preservation for older participants during economic growth (i.e., adopting a higher value of θ) and benefit stability during downturns (i.e., adopting a lower value of θ). Adaptive contribution rates could temporarily increase to stabilize the plan during financial stress. Additionally, provisions accounting for longevity trends can further enhance flexibility and fairness.

However, implementing such mechanisms requires careful consideration of legal, administrative and communication challenges. Stakeholder buy-in is essential to address potential resistance and foster trust in the plan's governance. Transparent communication about trade-offs and expected outcomes can build participant confidence and support.

8. Conclusion

Target benefit plans represent a promising approach to balancing the dual objectives of benefit stability and intergenerational equity. Through careful design and implementation, TBPs can address the complex challenges posed by varying market conditions, shifting demographics and stakeholder expectations. This paper has explored the implications of different benefit adjustment mechanisms and investment strategies, highlighting the trade-offs inherent in achieving stability and equity. By adopting flexible and forward-looking design elements, TBPs can remain effective and relevant in the face of economic and demographic changes.

A key insight from this study is the pivotal role adjustment mechanisms and investment strategies play in shaping TBP outcomes. Dynamic mechanisms and intermediate adjustment parameters present viable pathways to enhancing both equity and stability. However, transparent governance and proactive stakeholder communication remain essential. The ability of TBPs to adapt to emerging challenges will ultimately determine their sustainability and fairness in the long term.

Opportunities for future research

The findings of this study open avenues for further research to refine and enhance TBP designs. Key areas for exploration include:

1. **Hybrid adjustment mechanisms**: Investigating the combination of dynamic and static mechanisms to improve both stability and equity.
2. **Behavioural responses to flexibility**: Exploring how participants respond to flexible mechanisms, such as temporary contribution increases or phased benefit reductions. Understanding behavioural tendencies can inform more participant-friendly designs.
3. **Sustainability under extreme scenarios**: Simulating TBP performance under severe demographic or economic shifts to identify stress points and discover resilient structures.
4. **Technological innovations**: Leveraging machine learning and real-time analytics to optimize adjustment parameters dynamically based on market data. These tools can enhance precision and adaptability in TBP management.

5. **Equity impacts of investment choices**: Assessing how different investment strategies affect wealth transfers between participant cohorts. This analysis can guide strategic investment decisions that align with the plan's objectives.

Addressing these areas can yield actionable insights to make TBPs more resilient, equitable and sustainable. By integrating advanced analytics, participant-centred design principles and innovative adjustment mechanisms, TBPs will be better positioned to meet the needs of diverse participant groups while navigating future uncertainties.

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Appendix A: Main features of the model plan, membership profile and assumptions

Model plan

The model plan used in this analysis mimics key aspects of the Royal Mail CDC scheme, providing a conceptual framework for studying intergenerational risk-sharing in target benefit plans. The primary features of the model plan are as follows:

- **Pensionable pay**: Defined as basic salary only.
- **Employee contributions**: Not required.
- **Employer contributions**: Fixed at 10.83% of pensionable pay, calibrated to achieve the target retirement benefit.
- **Target retirement benefit**: 1/60th of pensionable pay for each year of service, adjusted annually for indexing until retirement.
- **Indexing approach**: Pre-retirement indexing is targeted to match the consumer price index, subject to plan revaluation rules.
- **Retirement age**: Normal retirement is set at age 65.
- **Form of benefit**: Lifetime pension with fixed-rate indexing; benefits are simplified as a lump-sum equivalent at retirement age for analysis purposes.
- **Ancillary benefits**: None included to focus exclusively on intergenerational risk dynamics.

This streamlined design facilitates the examination of intergenerational cross-subsidies without the complexities of post-retirement longevity and investment risks.

Membership profile

The membership profile of the model plan is intentionally simplified to highlight the dynamics of intergenerational risk sharing. Key characteristics include:

- **Generational structure**: Comprises 120 distinct cohorts, with one generation joining annually.
  - The first generation entered the plan 40 years ago (time 0).
  - The current cohort (40th generation) joined at time 39 (present time).
  - The final cohort (120th generation) will join at time 119.
- **Size and entry**: Each generation consists of 100 members, all joining the plan at age 25.
- **Membership phases**:
  - **Growing phase**: Years 0-39, during which membership expands.

    - **Stationary phase**: Years 40-119, with stable membership of 4,000 active members.
    - **Declining phase**: Years 120-159, marked by no new entrants and a diminishing active population.
- **Salary and growth**:
    - Initial salary is $50,000 for all members joining the plan at time 39.
    - Salaries grow by 3% annually, outpacing inflation and aligning with assumed economic growth.
- **Demographic assumptions**: All members are assumed to survive until age 65, ensuring a uniform analysis of benefit accrual and distribution.

The simplified membership model is designed to isolate and emphasize the intergenerational effects of benefit adjustment mechanisms, providing a clear understanding of how TBPs distribute risks and rewards across generations.

Economic and demographic assumptions

The analysis relies on static economic and demographic assumptions to provide a clear and consistent framework for evaluating the model plan:

- **Investment return**: The expected long-term return on the pension fund is 6% per annum, net of expenses. This reflects a balanced portfolio with significant exposure (50%) to equities and other risky assets.
- **Salary growth**: Annual salary increases are set at 3%, reflecting a consistent growth rate in line with economic expectations.
- **Inflation**: The target inflation rate is 2% per annum, providing a baseline for pre-retirement indexing.
- **Mortality**: Post-retirement mortality is based on a standard mortality table, resulting in a life annuity factor of 15 at age 65. This factor is derived using a 6% discount rate and a 2% annual indexing assumption.
- **Funding assumptions**: The employer contribution rate of 10.83% is calculated to provide the target retirement benefit for a member joining the plan at age 25 and retiring at 65, under the assumed economic and demographic conditions.
- **Income replacement ratio**: At retirement, the target income replacement ratio is estimated to be 56.6% of the final-year pay for members.

Appendix B: Economic scenario generator

This appendix outlines the economic scenario generator utilized in this study, as originally described in *A Stochastic Analysis of Policies Related to Funding of Defined Benefit Pension Plans* (Ma, 2023). The economic scenario generator simulates investment scenarios in the form of annual time series for long-term bond yields and equity returns.

Bond yield model

The economic scenario generator is based on the Vasicek model, which captures the evolution of long-term bond yields using a continuous mean-reverting process. The dynamics of the bond yield are governed by the following stochastic differential equation:

$$dy_t = \theta(\mu - y_t)dt + \sigma_y dW_t$$

where

- y_t is the instantaneous bond yield at time t;
- θ is the rate of reversion to the mean;
- μ is the long-run mean of the process;
- σ_y is the standard deviation of the bond yield process; and
- W_t is a standard Brownian motion.

In a discrete time setting, the mean-reverting process is approximated as:

$$y_{t+1} = y_t + \theta\,(\mu - y_t) + \varepsilon_y(t+1)$$

where $\varepsilon_y(t+1)$ represents a random shock at time $t+1$. These random shocks, $\{\varepsilon_y(t), t = 1, 2, \dots\}$, are independent and normally distributed with mean zero and variance σ_y^2, i.e., $\varepsilon_y(t) \sim i.i.d.N(0, \sigma_y^2)$.

Fixed income investments

The price process for fixed income investments (e.g., bonds) is modelled as:

$$BP_{t+1} = BP_t \cdot e^{y_t + D(t+1)\cdot(y_t - y_{t+1})}$$

where

- BP_t is the bond price at time t;
- $D(t+1)$ is the duration of the fixed income portfolio at time $t+1$, calculated as $h(t+1) \cdot D^L(t+1)$, where $h(t+1)$ is the hedge ratio, and $D^L(t+1)$ is the duration of the plan liabilities at time $t+1$.

Equity investments

For return-seeking investments (e.g., equities), the price process is given by:

$$EP_{t+1} = EP_t \cdot e^{y_t + ERP + \varepsilon_e(t+1)}$$

where $\varepsilon_e(t) \sim i.i.d. N(0, \sigma_e^2)$, σ_e is the standard deviation of the random shocks in equity returns, and ERP represents the equity risk premium.

The random variables $\varepsilon_y(t)$ and $\varepsilon_e(t)$ are assumed to be independent.

Parameter specification

The parameters for the bond yield model are calibrated based on historical data for federal bond yields, as reported in the *Report on Canadian Economic Statistics 1924–2020* by the Canadian Institute of Actuaries (Canadian Institute of Actuaries, 2021):

- $\mu = 0.0493$
- $\theta = 0.0194$
- $\sigma_y = 0.0076$

To reflect a neutral environment, the initial bond yield is set at 4%.

For the equity price model, the following values are used:

- equity risk premium: $ERP = 0.05$
- standard deviation: $\sigma_e = 0.15$

This parameterization ensures consistent simulation of economic scenarios for the purposes of this study.

Appendix C: Implementation of optimization in Python

In Section 5.2, we demonstrate how to find the optimal adjustment parameter (θ) that balances benefit stability and equity. This appendix provides the Python code using **scipy.optimize.minimize** with the **L-BFGS-B** method to find the adjustment parameter θ that minimizes the equity and stability index (λ).

The **L-BFGS-B** (Limited-memory Broyden–Fletcher–Goldfarb–Shanno with bound constraints) method is a popular optimization algorithm used for solving large-scale, non-linear optimization problems with bound constraints. It is an extension of the **L-BFGS** algorithm, which is a limited-memory version of the **BFGS** method, a quasi-Newton optimization technique. The key feature of **L-BFGS-B** is its ability to handle bound constraints on variables, making it suitable for problems where the solution must lie within specific bounds (Nocedal & Wright, 2006).

Below is the Python code used for optimization.

```
.. code:: ipython3

    import numpy as np
    import pandas as pd
    from scipy.interpolate import CubicSpline
    from scipy.optimize import minimize

    # Data
    adjustment_parameters = np.array([0, 0.2, 0.4, 0.5, 0.6, 0.8, 1.0])
    mean_std_dev_BPRs = np.array([0.011, 0.0373, 0.0562, 0.065, 0.0737, 0.0922, 0.1131])
    std_dev_CWRs = np.array([0.6108, 0.1153, 0.0244, 0.0179, 0.0269, 0.0443, 0.056])

    # Interpolate using cubic splines
    cs_mean_std_dev_BPRs = CubicSpline(adjustment_parameters, mean_std_dev_BPRs)
    cs_std_dev_CWRs = CubicSpline(adjustment_parameters, std_dev_CWRs)

    # Function to calculate the objective function
    def objective_function(x, w):
        if x < 0 or x > 1:
            return np.inf  # Out of bounds, return a high value
        mean_std_dev_BPRs_value = cs_mean_std_dev_BPRs(x)
        std_dev_CWRs_value = cs_std_dev_CWRs(x)
        return w * mean_std_dev_BPRs_value + (1 - w) * std_dev_CWRs_value

    # Weights for w
    weights = [0.0, 0.2, 0.4, 0.5, 0.6, 0.8, 1.0]

    # Results list to store the data
    results = []

    # Perform optimization for each weight
    for w in weights:
        result = minimize(objective_function, x0=0.5, args=(w,), bounds=[(0, 1)],
method="L-BFGS-B")
        optimal_adjustment_parameter = result.x[0]
        optimal_F_value = result.fun
        mean_std_dev_BPRs_value = cs_mean_std_dev_BPRs(optimal_adjustment_parameter)
        std_dev_CWRs_value = cs_std_dev_CWRs(optimal_adjustment_parameter)
        results.append({
            "Weight (w)": w,
            "Adjustment Parameter": optimal_adjustment_parameter,
            "Mean Std Dev BPRs": mean_std_dev_BPRs_value,
            "Std Dev CWRs": std_dev_CWRs_value,
            "Optimal Function Value": optimal_F_value
        })

    # Convert results to a DataFrame
    results_df = pd.DataFrame(results)
```

```
    # Save results to CSV
    file_path = r"C:\Users\chun_\Documents\From Google Drive\CDC_TBP\Type 3
adjustment\optimization_results.csv"
    results_df.to_csv(file_path, index=False)

    print(results_df)
```

Explanation of the code

- The code uses numpy and pandas for numerical operations and data handling.
- The CubicSpline function from scipy.interpolate is used to create smooth interpolations of the data.
- The minimize function from scipy.optimize is used to perform the optimization with the L-BFGS-B method.
- The results are stored in a pandas DataFrame and saved to a CSV file for further analysis.

Appendix D: Peer Review Report on "Balancing Stability and Equity in Target Benefit Plans: A Novel Benefit Adjustment Mechanism"

By Robert L. Brown, FCIA
January 2025

Brief Description of Paper

Chun-Ming (George) Ma's paper, *Balancing Stability and Equity in Target Benefit Plans: A Novel Benefit Adjustment Mechanism*, explores the challenges of balancing benefit stability and intergenerational equity in Target Benefit Plans (TBPs). TBPs, also known as collective defined contribution plans, aim to provide stable retirement income by pooling demographic and investment risks. However, the trade-off between benefit stability and intergenerational equity, particularly in volatile markets, remains a significant challenge. Ma's paper builds on his previous work, *Balancing Act: Exploring Intergenerational Risk in Target Benefit Plans (Ma, 2024),* by proposing a novel benefit adjustment mechanism with a parameter θ to better manage benefit volatility and intergenerational wealth transfers. The paper introduces two key risk measures—**Benefit Stability Risk Measure** and **Wealth Risk Measure**—and a composite **Equity and Stability Index** to evaluate trade-offs. Additionally, the paper analyzes the impact of different investment strategies on benefit outcomes and offers actionable insights for designing sustainable and equitable TBPs.

Research Problem

The research problem addressed in Ma's paper is highly relevant in the context of modern pension design. TBPs have gained attention as an alternative to traditional defined benefit (DB) and defined contribution (DC) plans, offering a middle ground by pooling risks among participants. However, the pooling of risks, particularly intergenerational risk-sharing, raises concerns about equity, as younger or mid-career participants may bear a disproportionate share of the adjustments needed to stabilize benefits for older participants nearing retirement. Ma's paper seeks to address this challenge by proposing a more balanced benefit adjustment mechanism that reduces the inequities inherent in current TBP designs, such as the UK Royal Mail Collective Defined Contribution (RM-CDC) scheme.

Ma's previous paper, *Balancing Act: Exploring Intergenerational Risk in Target Benefit Plans*, laid the groundwork for this study by examining the accumulation phase of the RM-CDC scheme and analyzing how intergenerational risk-sharing operates under different market scenarios. The current paper builds on this foundation by delving deeper into the balance between benefit stability and intergenerational equity, proposing a novel adjustment mechanism that aims to achieve a fairer distribution of risks across generations.

Approach Taken in Review

This review is based on:

1. My understanding of the *CIA Task Force Report on Target Benefit Plans (2015)*, which provides a comprehensive overview of the challenges and opportunities associated with TBPs.

2. My broad knowledge of pension plan designs worldwide, particularly target benefit plans or collective defined contribution plans.
3. My reading of Ma's previous paper, *Balancing Act: Exploring Intergenerational Risk in Target Benefit Plans*, which provides the theoretical foundation for the current study.

I did not attempt to replicate the author's calculations, but I reviewed the reasonableness and appropriateness of the assumptions and methods used in the research. The methodology appears robust, and the assumptions are well-justified, making the findings credible and relevant to the field of actuarial science.

Main Contributions of the Paper

Ma's paper makes several significant contributions to the field of pension design and actuarial science:

1. **Novel Benefit Adjustment Mechanism**: The paper proposes a new benefit adjustment mechanism with parameter θ, which allows for more control over benefit payout volatility and intergenerational wealth transfers. This mechanism aims to distribute the effects of funding surpluses or deficits more equitably across all participants, reducing the burden on specific cohorts, particularly mid-career participants.
2. **Introduction of Key Risk Measures**: The paper introduces two key risk measures which provide a robust framework for analyzing the trade-offs between benefit stability and equity in TBPs:
    a. Benefit Stability Risk Measure: Quantifies the volatility of benefit payouts.
    b. Wealth Risk Measure: Evaluates the extent of intergenerational wealth transfers.
3. **Equity and Stability Index**: The paper introduces a composite measure, the Equity and Stability Index, which helps determine the optimal adjustment parameter for balancing benefit stability and intergenerational equity. This index provides a quantitative tool for plan sponsors and policymakers to evaluate and compare the outcomes of different plan designs under varying market conditions.
4. **Actionable Insights for TBP Design**: The paper offers practical insights for designing TBPs that align with the plan's objectives and the risk tolerance of its stakeholders. By analyzing the impact of different investment strategies (aggressive, balanced, and conservative), the paper provides a comprehensive understanding of how investment decisions influence both benefit stability and equity.

Analysis Methodology

Ma's paper employs a rigorous and well-structured methodology to analyze the trade-offs between benefit stability and intergenerational equity in TBPs. The key components of the methodology include:

1. **Streamlined Model TBP**: The paper uses a simplified model TBP, based on the UK Royal Mail CDC scheme, to isolate and emphasize the intergenerational effects of benefit adjustment mechanisms. The model includes a membership profile with 120 distinct cohorts, each consisting of 100 members, and assumes a fixed employer contribution rate of 10.83%.
2. **Stochastic Simulations**: The paper uses stochastic simulations, with investment scenarios generated from an economic scenario generator (ESG), to simulate benefit outcomes under the TBP with various adjustment mechanisms. The simulations cover a range of market conditions and compare the outcomes with those of Individual Defined Contribution (IDC) plans.
3. **Trade-off Analysis:** The paper analyzes the trade-offs between benefit stability and intergenerational equity by introducing two key metrics—the Benefit Payout Ratio (BPR) and the Collective Wealth Ratio (CWR). These metrics allow for a comprehensive evaluation of how different adjustment mechanisms and investment strategies influence benefit outcomes.
4. **Optimization of Adjustment Parameters:** The paper employs an optimization process to determine the optimal adjustment parameter (θ) that balances benefit stability and intergenerational equity. The Equity and Stability Index (λ) is used as the objective function, with the goal of minimizing λ to achieve the desired balance.
5. **Implications of Investment Strategies**: The paper also considers the implications of different investment strategies (aggressive, balanced, and conservative) on benefit outcomes. This analysis provides valuable insights for plan decision-makers, highlighting the trade-offs between higher returns and increased volatility.

Overall, the methodology is robust and well-suited to address the research problem. The use of stochastic simulations and optimization techniques ensures that the findings are both credible and actionable.

Assessment of the Paper

1. **Clarity of Research Problem and Significance**: The research problem is clearly stated and highly significant. The paper addresses a critical challenge in pension design—balancing benefit stability and intergenerational equity—and proposes a novel solution that has the potential to improve the sustainability and fairness of TBPs.
2. **Interest to CIA Members and the Public**: The topic of the paper is likely to be of great interest to CIA members, particularly those involved in pension design and risk management. The public, especially those concerned with retirement security, may also find the paper's insights valuable, as it addresses a key issue in modern pension systems.
3. **Clarity of Key Findings:** The key findings are clearly stated and well-supported by the analysis. The paper demonstrates how the proposed adjustment mechanism can achieve a more equitable distribution of risks across generations, while also providing actionable insights for designing sustainable TBPs.

4. **Accessibility and Writing Quality:** The paper is well-written and accessible to its intended audience. The use of clear explanations, illustrative examples, and visual aids (e.g., figures and tables) enhances the readability of the paper.
5. **Dissemination of Findings:** The findings of the paper are highly relevant and should be disseminated widely. The paper's insights could inform policy discussions and guide the design of future TBPs, both in Canada and internationally.

Opportunities for Further Research

The paper concludes with several promising avenues for further research, including:

1. **Hybrid Adjustment Mechanisms**: Investigating the combination of dynamic and static mechanisms to improve both stability and equity.
2. **Behavioral Responses to Flexibility**: Exploring how participants respond to flexible mechanisms, such as temporary contribution increases or phased benefit reductions.
3. **Sustainability under Extreme Scenarios**: Simulating TBP performance under severe demographic or economic shifts to identify stress points and discover resilient structures.
4. **Technological Innovations**: Leveraging machine learning and real-time analytics to optimize adjustment parameters dynamically based on market data.
5. **Equity Impacts of Investment Choices**: Assessing how different investment strategies affect wealth transfers between participant cohorts.

These research opportunities have the potential to further refine and enhance TBP designs, making them more resilient, equitable, and sustainable in the face of future uncertainties.

References

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- Ma, C.-M. (2017, April). An Actuarial Balance Sheet Approach to Assessing Sustainability of Target Benefit Plans. Canadian Institute of Actuaries. **https://www.cia-ica.ca/publications/217045e/**
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Conclusion

Chun-Ming (George) Ma's paper makes a significant contribution to the field of pension design by proposing a novel benefit adjustment mechanism that balances benefit stability and intergenerational equity in TBPs. The paper's rigorous methodology, clear findings, and actionable insights make it a valuable resource for actuaries, policymakers, and stakeholders involved in the design and management of retirement plans. The paper's conclusions and recommendations are well-supported, and the proposed adjustment mechanism has the potential to improve the sustainability and fairness of TBPs in dynamic economic environments.

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